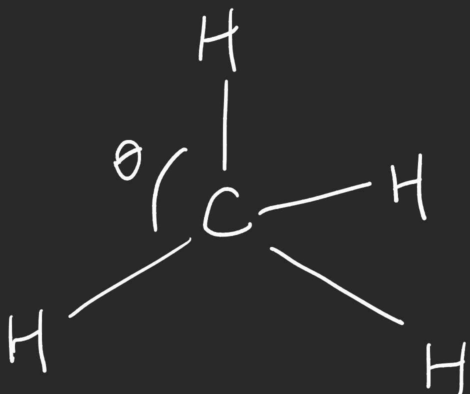
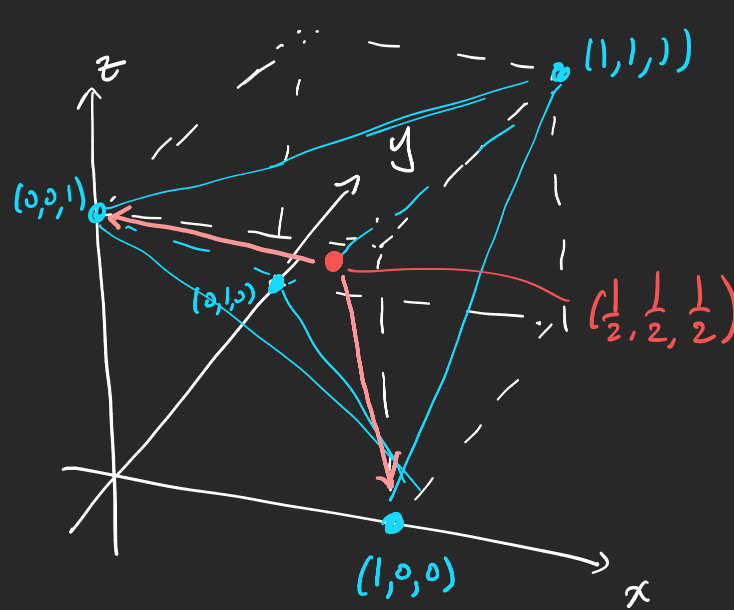
Minor correction to Lec 3's posted slides:

"The component (scalar projection) is, up to sign, the magnitude of the vector projection."

Example §12.3 #57



what is θ?



$\vec{u} =$ $= \text{head} - \text{tail} = \langle -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \rangle$

$\vec{v} =$ $= \langle \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \rangle$

$$\cos\theta_{\vec{u},\vec{v}} = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{(-\frac{1}{2})(\frac{1}{2})+(-\frac{1}{2})(-\frac{1}{2})+(\frac{1}{2})(-\frac{1}{2})}{\sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}\sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}}$$

$$= \frac{-1/4}{3/4} = -1/3$$

$$\theta_{\vec{u},\vec{v}} = \cos^{-1}(-1/3) \approx 109.5^\circ$$

There is also a formula

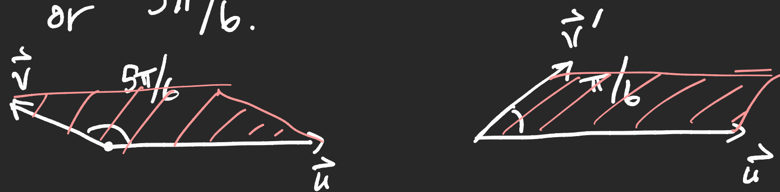
$$\sin\theta_{\vec{u},\vec{v}} = \frac{|\vec{u}\times\vec{v}|}{|\vec{u}||\vec{v}|}, \qquad (*)$$

Why not use this formula to compute $\theta_{\vec{u},\vec{v}}$?

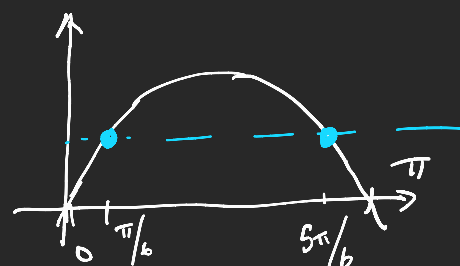
A: If for example I compute

$$\sin\theta = 1/2$$

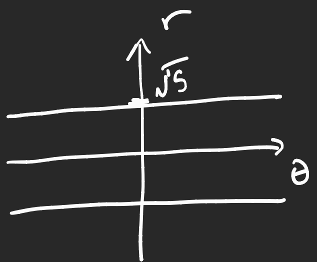
this does not tell me whether $\theta = \pi/6$ or $5\pi/6$.

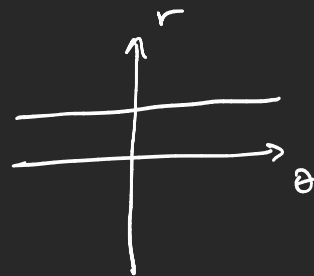


(*) cannot distinguish between these two pictures.

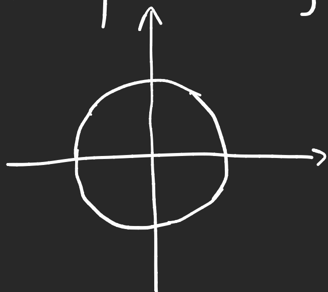


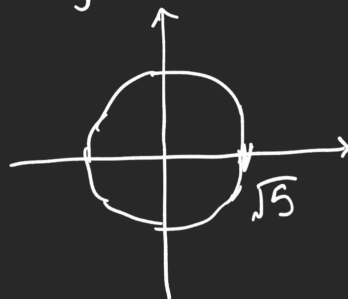
$r^2 = 5$ vs $r = \sqrt{5}$





same picture in xy-plane though:





similar examples: $\theta = \frac{\pi}{4}$ vs $\theta = \frac{5\pi}{4}$